Provincial Standards:
Mathematics

Grades 3, 6, and 9

1993

Contents

Introduction

Background

This document provides standards of performance in mathematics for the end of Grades 3, 6, and 9. Future revisions to *The Common Curriculm*[1] may result in revisions to the Standards of Performance. In addition, the Royal Commission on Learning may recommend changes.

The document is the result of close collaboration among mathematics teachers, Ministry of Education and Training officials, and representatives of the three mathematics associations in Ontario; it also incorporates materials from school boards in Ontario and other jurisdictions and countries. The Ministry of Education and Training is pleased to acknowledge the co-operation and assistance of the teachers, researchers, and co-ordinators who have contributed to the development of these standards.

Figure 1 shows the purposes of and relationships among outcomes, standards, assessment, and reporting.

Figure 1: Outcomes, Standards, Assessment, and Reporting

Determining Outcomes

- Outcomes state general results expected.
  - *The Common Curriculm* determines the knowledge, skills, and values expected for all.

Setting Standards

- Standards state the expected range of student achievement for the outcomes.
  - The province will provide clear standards in language and mathematics.

Assessing Performance

- Performance will be assessed against the provincial standards in language and mathematics (Grades 3, 6, and 9).
  - Classroom teachers will have common provincial standards as a basis for evaluating and reporting student achievement.
  - School boards and the province will use the same standards for board-wide and provincial testing.

Reporting Results

- Results will be reported in relation to the provincial standards in language and mathematics (Grades 3, 6, and 9).
  - Classroom teachers will, where appropriate, report student achievement in relation to the standards.
  - School boards and the province will report school-wide, school board, and provincial performance in relation to the standards.

1. Ministry of Education and Training, Ontario, *The Common Curriculum, Grades 1–9* (Toronto: Ministry of Education and Training, Ontario, February 1993).

A Framework for Assessment

Assessment takes place within a particular context or framework of expectations. In this document, the Ministry of Education and Training seeks to provide such a framework using the organization and format described below.

The document has three main sections. The first of these, "Key Components of Mathematics Learning", discusses the fundamental skills of communication, reasoning, and problem solving as integral components of mathematics learning.

For the purposes of the discussion in sections two and three, the concepts and operations taught in the mathematics program are grouped into six specific mathematical areas, or "strands". These are:
- problem solving and inquiry;
- number sense and numeration;
- geometry and spatial sense;
- measurement;
- patterning and algebra;
- data management and probability.

In the second section, "Strands of Mathematics", important characteristics of each strand and the appropriate methodology for teaching it are discussed.

The third section, "Standards of Performance", outlines in detail a framework that can be used to measure students' performance in each strand of the program at the end of Grades 3, 6, and 9. The components of this framework are as follows:

1) *Outcomes*. For each strand, the expected outcomes from *The Common Curriculum* are stated, identifying the mathematical understanding students should have acquired and/or the operations they should be able to perform.

2) *Interpretation of Outcomes*. A list of specific understandings and/or skills related to the expected outcomes for each strand is also provided.

3) *Standards of Performance*. For each strand, four "Standards of Performance" statements are provided. These are general descriptions of the degrees of proficiency students exhibit.

4) *Sample Performance Indicators*. A list of "Sample Performance Indicators" is given for each level of performance. These offer specific examples of what students know and can do at each of the four levels. While efforts have been made to provide substantial lists of performance indicators, the lists are not intended to be exhaustive or restrictive.

It should be noted that problem solving, which is a central focus of mathematics learning, is treated both as a strand of the mathematics program and as an integral part of all other strands. The Sample Performance Indicators described for problem solving are applicable to all grades.

Figure 2: Framework for Assessment of Performance in Mathematics

Level	Strands						
	Problem Solving and Inquiry	Number Sense and Numeration	Geometry and Spatial Sense	Measurement	Patterning and Algebra	Data Management and Probability	
4 Superior Performance							Students performing beyond expected range of standards set for the grade.
3 Proficient Performance							*Provincial Standards* Students performing within the expected range set for the grade.
2 Adequate Performance							
1 Limited Performance							Students not yet performing within the standard for the grade.

Standards of Performance

Descriptions of performance at four levels are provided for each strand at the end of Grade 3, Grade 6, and Grade 9, as shown in figure 2. Each description includes:
- a word describing the quality of the overall performance (e.g., "limited", "adequate", "proficient", "superior");
- a general description of what students can do in the mathematical area;
- a list of Sample Performance Indicators providing specific examples of what students can do in the area.

What the Standards Mean

Students do not learn things in the same order or at the same rate. In addition, the nature and context of performance will be very different for an eight-, eleven-, or fourteen-year-old. The four levels demonstrate the range of performance for each of the three identified grades. It is important to recognize that the levels do not represent a simple continuum – for example, level-three performance in Grade 3 is not equivalent to level-one performance in Grade 6.

Of the four categories described, levels two and three are considered to be within the expected standard for student performance in Ontario (Adequate and Proficient). Performance described as Limited is below the expected standard; performance described as Superior is beyond the expected standard.

It is unlikely that the performance of any one student will fall exclusively into a single category; rather, a student's level of performance will vary according to the strand of mathematics. **It is important to recognize that the Standards of Performance represent descriptions of student performance, not descriptions of students.**

The Standards of Performance describe high but reasonable expectations. An effort has been made to blend observations of current performance with desired performances, in order to set standards that are future-oriented but realistic.

How the Standards Can Be Used

The Provincial Standards for Mathematics are broad standards for the province against which to assess students' learning. They set worthwhile goals for all, by describing expected provincial levels of performance. Their primary aim is to make a clear statement to teachers, parents, and students about expected results.

Teachers may use the more specific outcomes as a basis for team planning in divisions. They may use statements of the Standards of Performance, together with the Sample Performance Indicators, to assess an individual student's work and to plan with the student and parents for improvement. As well, teachers may use the standards as a reference in reporting student performance to parents.

Parents may use the Provincial Standards of Performance to assess their children's performance. School boards and the Ministry of Education and Training may use the Standards of Performance as a basis for program evaluation.

Assessment and Evaluation in the Classroom

In evaluating student performance in relation to the Standards of Performance described in this document, it is important for the teacher to use a wide range of assessment strategies. A thorough description of alternative assessment strategies is included in Appendix B. **During the 1993-94 school year, the Ministry of Education and Training, in co-operation with Ontario school boards and provincial mathematics associations, is developing a resource bank of high-quality performance assessment materials for Grades 3, 6, and 9 mathematics.**

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Key Components of Mathematics Learning

Communication, reasoning, and problem solving are central and essential components of mathematics learning. The following descriptions of these components apply throughout the Formative and Transition Years. Relevant outcomes from *The Common Curriculum* are identified for each component.

The introductory discussions of each component are adapted from the document *Focus on Renewal of Mathematics Education* (1993), prepared by the Ontario Association for Mathematics Education (OAME) and the Ontario Mathematics Coordinators Association (OMCA).

Communication

Mathematics is a language that must be made meaningful to students if they are to communicate mathematically and apply mathematics productively. Students' general communicative ability helps them to understand the language of mathematics. Their everyday language is a bridge that enables them to translate their informal, intuitive notions into the abstract language and symbolism of mathematics. Communication also plays a key role in helping students to make important connections among physical, pictorial, graphic, symbolic, verbal, and mental representations of mathematical ideas and, as a corollary, to gain a sense of the value of mathematics as a type of communication system and as a tool. When students understand that one representation can describe many situations, and that some ways of representing a problem are more helpful than others, they begin to understand the power, flexibility, and usefulness of mathematics.

It is important for students to have opportunities to "talk mathematics". Interacting with classmates helps students to construct knowledge, learn other ways to think about ideas, and clarify their own thinking. Writing about mathematics – for example, describing how a problem was solved – also helps students to clarify their thinking and develop deeper understanding.

Students need to be actively involved in "doing mathematics". The activities of exploring, investigating, describing, and explaining mathematical ideas promote the development of communication skills. Teachers facilitate this development when they pose probing questions and invite students to explain their thinking. Teachers can also assess students' knowledge and insight by listening and observing. The idea that students should learn mathematics through activities that have relevance to their own lives is implicit in this discussion. That is, students need to see mathematics as an effective means for understanding, describing, and responding to the world around them.

As students' abilities to reason abstractly continue to develop, so too should their abilities to communicate mathematically.

Communication Outcomes From *The Common Curriculum*

Grade 3

Students will:

- read, listen, observe, represent, construct, discuss, write, diagram, chart, and graph in order to investigate and acquire skills and knowledge in mathematics, science, and technology. (p. 80)
- read, discuss, and follow simple instructions in mathematics, science, and technology and communicate them using appropriate terminology. (p. 80)
- demonstrate the ability and willingness to work independently and with others, and be willing to engage in class discussions about co-operation and sharing. (p. 81)
- recognize and use alternative representations of the same idea. (p. 89)

Grade 6

Students will:

- create and/or use models, symbols, pictures, graphs, charts, diagrams, and terminology in order to investigate and acquire skills and knowledge in mathematics, science, and technology. (p. 80)
- read, discuss, follow, and evaluate instructions in mathematics, science, and technology, and communicate them using appropriate terminology, models, and symbols. (p. 80)
- demonstrate the ability and willingness to work independently and with others, and to join in developing rules of group conduct with peers. (p. 81)
- describe, in precise language, their impressions of and reactions to the aesthetic qualities of manufactured and natural objects, systems, ideas, and environments. (p. 88)
- recognize and use alternative representations of the same idea. (p. 89)

Grade 9

Students will:

- use a variety of methods, technologies, models, checklists; sketches, graphs, and charts; oral, written, and pictorial symbols; and technological devices in order to investigate, acquire skills and knowledge, and communicate effectively in mathematics, science, and technology. (p. 80)
- acquire information by reading mathematical, scientific, and technological journals, manuals, and books and consulting databases; ask clarifying or extending questions; and apply the information accurately. (p. 80)
- demonstrate the ability and willingness to work independently and with others, and to explain the importance of collaboration for the advancement of knowledge in mathematics, science, and technology. (p. 81)
- articulate and defend, in precise language, their judgements about the aesthetic qualities of objects, systems, ideas, and environments. (p. 88)
- recognize, compare, and use alternative representations of the same idea. (p. 89)
- use advanced features of word-processing, telecommunications, database-management, and spreadsheet technologies to organize, analyse, interpret, and communicate information in a fashion appropriate to the intended audience. (p. 94)

Reasoning

Reasoning is fundamental to the knowing and doing of mathematics. Our goal is to help students develop the belief that they have the power to do mathematics and that they have control over their own success or failure. Students' autonomy develops as they gain confidence in their ability to reason and to justify their thinking. Conjecturing and demonstrating the validity of conjectures are the essence of the creative act of doing mathematics. An emphasis on reasoning must pervade all classroom activity.

Students need a reasonable amount of time and varied experiences to construct convincing arguments during problem–solving activities and to evaluate the arguments of others. A climate should be established in the classroom that places critical thinking at the heart of instruction. Both the teacher's and the students' statements should be open to questions, reaction, and elaboration from others in the classroom. For such a positive spirit of inquiry to prevail, all members of the class need to express genuine respect and support for one another's ideas.

The development of the ability to reason logically is tied to the intellectual and verbal development of students. Students begin as concrete thinkers who depend on a physical or concrete context to support their reasoning. Increasingly through the Formative and the Transition Years, students will become more capable of formal reasoning and abstraction. However, even the most advanced students may relate to the use of concrete materials to support their reasoning; this is especially true for spatial reasoning.

To develop their mathematical reasoning ability, students must have opportunities to explore, speculate, validate, and convince others. A classroom that provides challenging mathematical experiences and appropriate materials and tools will best meet this goal.

Problem Solving

As adults, students will use their mathematics knowledge and skills to help them find solutions to problems in their daily lives and to make decisions based on those solutions. Problem solving should be the central focus of the mathematics curriculum. It is the process by which students experience the power and usefulness of mathematics in the world around them. It is a method of inquiry and application that should be interwoven with the other five strands of mathematics to provide a consistent context for learning and applying mathematics.

Classrooms should provide a comprehensive and rich approach to problem solving in a climate that encourages and supports students' efforts. Ideally, students should share their thinking and strategies with other students and with teachers. They should learn a variety of ways of representing and solving problems. In addition, they should value the process of solving problems as much as they value the solutions. Students should have many experiences that involve posing problems based on real-world activities, on organized data, and on equations.

Although problems will usually be based on concrete and empirical situations, the program should strike a balance between problems that apply mathematics to the real world and problems that arise solely from the investigation of mathematical ideas. The mathematics curriculum should also engage students in some problems that take an extended effort to solve. These

problems may be part of group projects that require students to use available technology and to undertake co-operative problem solving and discussion.

Computers and calculators are powerful problem–solving tools. The power to compute rapidly, or to investigate relationships among variables using spreadsheets and graphing programs, can help students to meet and even seek out mathematical challenges with increasing resourcefulness and independence.

Problem-Solving Outcomes From *The Common Curriculum*

Grade 3

Students will:
- understand and follow the procedures necessary to ensure an honest investigation or inquiry. (p. 80)
- understand and explain why it is important to be accurate, thorough, and persistent when conducting an inquiry. (p. 81)
- find multiple solutions for a problem and respect and accept other people's solutions. (p. 81)
- identify relationships among mathematics, science, and technology in order to conduct interdisciplinary inquiries at the home, school, and local community level. (p. 89)
- ask questions about the world around them and discuss the validity of answers. (p. 94)
- follow the steps used in a standard scientific/technological inquiry method. (p. 95)
- demonstrate a beginning understanding of problem–solving strategies in mathematics, science, and technology. (p. 96)
- recognize and solve personal problems and simple problems related to a familiar environment by applying thinking and communication skills acquired through mathematics, science, and technology activities. (p. 96)
- work individually and in groups to identify problems on issues of concern to them personally or to the community and investigate and make judgements about the problems or issues. (p. 97)

Grade 6

Students will:
- detect biases, omissions, and distortions in inquiries and learning materials and understand and follow procedures that will ensure the honesty and integrity of an inquiry. (p. 80)
- conduct an accurate and thorough inquiry with persistence and creativity and explain the importance of those qualities in investigations using mathematical, scientific, and technological methods. (p. 81)
- recognize that there are a variety of solutions for many problems, assess the validity and quality of different solutions, determine the most appropriate solution, and respect and accept other people's solutions. (p. 81)
- identify relationships among mathematics, science, and technology in order to conduct interdisciplinary inquiries at the community level. (p. 89)
- ask questions about and investigate their immediate environment and the wider world, discuss the answers, and communicate the results to others. (p. 94)
- use scientific/technological methods during inquiry processes, retracing and repeating steps as required. (p. 95)

- use problem–solving strategies in mathematics, science, and technology. (p. 96)
- use a variety of strategies to pose and solve everyday problems with a mathematical, scientific, or technological dimension, and evaluate and justify their conclusions. (p. 96)
- use scientific, mathematical, and technological methods of inquiry to investigate, understand, and describe local areas and issues. (p. 96)
- work individually and in groups to investigate, make decisions about, and reflect upon personal, societal, global, and environmental problems. (p. 97)

Grade 9

Students will:
- understand, value, and follow procedures that will ensure the honesty and integrity of an inquiry and be able to explain why these qualities are important in scientific and technological investigations. (p. 80)
- conduct an accurate, thorough inquiry with persistence and creativity and explain why these and other qualities are required in investigations using mathematical, scientific, and technological methods. (p. 81)
- value the fact that problems may be solved in different ways and respect and accept other people's solutions. (p. 81)
- identify relationships among mathematics, science, and technology in order to conduct interdisciplinary inquiries at the regional, national, and global level. (p. 89)
- ask questions and pursue answers through independent inquiry and research, and communicate their findings in a variety of formats and to a variety of audiences. (p. 94)
- understand the importance to science and technology of scientific/technological methods of inquiry, use such methods to conduct a personal inquiry, and explain the inquiry process used to others. (p. 95)
- use a variety of their own and generally accepted problem–solving strategies in mathematics, science, and technology. (p. 96)
- use a variety of strategies to pose and solve a wide range of everyday problems drawn from relevant experiences across the curriculum and having a mathematical, scientific, or technological dimension. (p. 96)
- use a variety of problem–solving strategies to investigate the capacity of the earth to sustain life and development. (p. 96)
- work individually and in groups to investigate, make decisions about, and reflect and act upon personal, societal, global, and environmental problems. (p. 97)

Strands of Mathematics

For purposes of organization of the Standards of Performance, the following strands of mathematics have been used:
- problem solving and inquiry;
- number sense and numeration;
- geometry and spatial sense;
- measurement;
- patterning and algebra;
- data management and probability.

With the exception of the strand "problem solving and inquiry", Standards of Performance for the end of Grade 3, Grade 6, and Grade 9 are provided for each strand. For "problem solving and inquiry", a single set of standards is provided that applies across the grades.

The following section presents a discussion of the nature of each strand, including relevant methodological and developmental considerations. The discussion is adapted from the document *Focus on Renewal of Mathematics Education*.

Problem Solving and Inquiry

Inquiry Learning

"Children arrive in school with open, inquiring minds. They are already familiar with the inquiry process through their experiences with play, and their most important need in the Primary years is to have opportunities to continue their natural inclination towards inquiry learning."[2]

Inquiry learning is natural to young children. Primary teachers are familiar with and use the seven main stages of the inquiry process (exploring, inquiring, predicting possibilities, planning and collecting, deciding, communicating, evaluating) to help students to become responsible for their own learning.

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2. Ministry of Education, Ontario, *Shared Discovery: Teaching and Learning in the Primary Years* (Toronto: Ministry of Education, Ontario, 1985), p. 9.

A Problem-Solving Model

Another form of the inquiry model is a multi-step process based on George Polya's research (understand the problem; make a plan; carry out the plan; look back; communicate the solution).[3] This model provides problem-solvers with a framework within which to tackle problems. It is recommended that this general model for mathematics not be directly introduced to most children before Grade 3. The model, however, can be useful as a further mental framework for Primary teachers as they plan the classroom environment, take advantage of incidental opportunities for learning, and pose questions to help students become more successful problem solvers. As students progress through the grades, direct instruction using this model should occur. A poster of the model should be prominent in the classroom for reference during instruction and problem–solving activities.

Problem Solving As a Focus

Students should solve problems on a regular and frequent basis. Problem solving should not be viewed or presented as an isolated unit of study; rather, it should provide the context in which mathematics skills and concepts are learned. In a classroom where problem solving is an integral part of instruction, there is an emphasis on having students explore a variety of different kinds of problems (with a special focus on problems where the solution is not immediately evident) and on encouraging students to share their thinking. In a supportive classroom setting, solving problems in different ways is applauded, the process is valued as much as the solution, and students are actively involved in posing problems based on real-world activities.

Problem Solving As a Process

Students should have opportunities to work both individually and co-operatively on problems that are open-ended, that involve assumptions and limitations, and for which multiple solutions may be possible. Typical arithmetic problems are restricted – there is usually one correct answer and one way to reach that answer. A program involving only arithmetical problem solving would present students with a very limited view of the problem–solving process.

Problem solving is a process that may involve starting and stopping, failure and success, consideration and rejection, testing and exploration. It is important for teachers to model the process of problem solving in the classroom. Students sometimes have the impression that successful problem solvers just know what to do and can come up with a solution right away. If the teacher models or thinks out loud while solving a problem, and encourages students to do the same, all students may acquire a better appreciation of the process.

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3. G. Polya, *How to Solve It: A New Aspect of Mathematical Method,* 2nd ed. (1945; Princeton, N.J.: Princeton University Press, 1973).

Number Sense and Numeration

Number Sense

Number sense has been defined as

> an intuition about numbers that is drawn from all the varied meanings of number. It has five components:
> 1. Developing number meanings ....
> 2. Exploring number relationships with manipulatives ....
> 3. Understanding the relative magnitudes of numbers ....
> 4. Developing intuitions about the relative effect of operating on numbers ....
> 5. Developing referents for measures of common objects and situations in their environment ....[4]

All students require purposeful classroom experiences that will assist them in developing number sense. Appropriate experiences would include opportunities for students to manipulate physical objects and use their own language to explain their thinking.

Students with good "operation sense" recognize situations in the real world where a particular arithmetical operation would be useful. They can also construct appropriate models and describe the steps of an operation, relationships among operations, and the effects of an operation on a pair of numbers.

Mental Mathematics and Computational Estimation

Students should develop mastery of basic one-digit computational facts by the end of Grade 6. To meet this goal, students require time to identify relationships between numbers and to develop efficient thinking strategies. Mastery is a critical component of fluency with paper-and-pencil and mental computation, as well as estimation.

Techniques of mental mathematics should be introduced gradually along with concepts of place value and paper-and-pencil computations. For example, the mental skill of adding numbers ending in zero, such as $20 + 30 + 60$, can be learned by a student who understands the place value and can add $2 + 3 + 6$.

After students have established a firm base in mental mathematics, the systematic teaching of strategies of computational estimation should occur. Computational estimation should include not only applications involving whole numbers and decimals, but also those involving fractions and per cent. In fact, early experiences in estimating with per cent and fractions help to establish related number sense.

It is essential for students to develop strong skills and strategies in mental mathematics and computational estimation, in order to judge the reasonableness of answers produced by a calculator or computer.

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4. Working Groups of the Commission on Standards for School Mathematics, *Curriculum and Evaluation Standards for School Mathematics* (Reston, Va.: National Council of Teachers of Mathematics, 1989), pp. 39-40.

Paper-and-Pencil Computations

Whole Numbers and Decimals

Many paper-and-pencil computations, such as long division, operations with large numbers, or calculating the square root of a number, have become less used in light of the availability of calculators and computers. Emphasis on estimation supported by competence in mental calculation will enhance a student's ability to cope in our technological society.

Students (and adults) may continue to need some facility with paper-and-pencil computations, however. The following guidelines are recommended as being reasonable **maximum** limits for this kind of computation **by the end of Grade 6**:

Addition	three four-digit numbers
Subtraction	two four-digit numbers
Multiplication	a two-digit number by a two-digit number
Division	a three-digit number by a one-digit number (including short division)

It is important for paper-and-pencil computational procedures to be introduced through the use of concrete materials, and for students to develop skill through practice in context. It is inappropriate to engage students in pages of isolated drill of computational skills.

In the Transition Years, students who have achieved the maximum required levels should be given opportunities to maintain them through practice in context; students who have not achieved the required levels should be given opportunities to extend their skills.

Fractions and Rationals

Concepts and operations with fractions should be introduced using concrete materials. Models such as physical materials and diagrams should be used to relate fractions to decimals, to find equivalent fractions, and to explore operations with fractions and decimals. Fraction symbols should be introduced only when students have developed the concepts and oral language to make the symbols meaningful.

In the Formative Years, the focus should be on developing fraction sense, a process that includes such things as representing fractions concretely, pictorially, and symbolically; identifying, generating, and applying equivalent fractions; estimating with fractions and mixed numbers; and ordering fractions and mixed numbers. Operations with fractions should be limited to addition and subtraction, and to multiplication involving a fraction of a whole number. Formal procedures should be introduced only after students have had significant opportunities to develop understandings using concrete materials. In operations with whole numbers, there is a recognized multi-stage process that leads students from the concrete to the abstract, or from base 10 materials to the formal algorithms. Similarly, as students explore fractions with concrete materials such as pattern blocks, the notion of addition and subtraction becomes apparent. For example, where a yellow hexagon equals one whole, a green triangle has a value of $\frac{1}{6}$. Simple counting of two triangles represents $\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$. If four green triangles are counted out to represent $\frac{4}{6}$, and one is removed, the operation of $\frac{4}{6} - \frac{1}{6} = \frac{3}{6}$ has been performed. It is important for students to have concrete experiences such as these before they attempt formal operations with fraction symbols.

It is recommended that, initially, simple denominators such as 2, 3, 4, 5, and 10 be used. As students gain more experience and skill in working with fractions, denominators such as 6, 8, and 12 can be included.

In the Transition Years, the focus should be on using fractions in the context of ratios, rates, and per cent. As well, students should extend their fraction sense to include skill in operations with fractions. (It should be remembered, however, that the most frequent real-life use of fractions involves estimation.) Signed fractions (rational numbers) can be introduced in Grade 9, but only simple calculations should be expected.

Use of the Calculator

In the Formative Years, calculators should be used for investigations, problem–solving experiences, and real-life applications in which the complexity of the numbers is greater than the computational skills of the students. In the Transition Years, calculators should be used except when the purpose of an activity is to develop paper-and-pencil computational skills.

When using calculators in all grades, the focus should be on the development of the ability to use the calculator appropriately and **effectively** and to judge the reasonableness of an answer produced. Students must learn to decide whether a calculator is needed for a particular computation, or whether mental mathematics is more appropriate. The calculator is a powerful aid to computation, but its use must not be allowed to interfere with the intelligent application of good number sense and numeration skills.

Decision Making in Computation

It is essential for all students to have access to a wide range of computational tools, including concrete materials, mental mathematics, estimation, paper-and-pencil procedures, and calculators. During the problem–solving process, students will decide which tool is most appropriate in a given set of circumstances.

Geometry and Spatial Sense

Spatial Sense

Spatial sense is the intuitive feel for one's surroundings and the objects in them. Students need a rich learning environment that contains a variety of geometric objects that they can manipulate in order to discover the geometric properties of objects and the relationships among them.

Communication Skills

Students need to develop and use a variety of communication skills in geometry. Geometry uses a formal language to describe objects, and their interrelationships and movements in space. In the Formative Years, however, teaching students the formal language of geometry is not as important as helping them to identify geometric properties and principles and to use pictures or everyday

language to explain their observations. In the Transition Years, the student's personal understandings can grow into a correct and effective use of the vocabulary of geometry. The teacher's role in language development is to blend the student's own language with the correct mathematical terminology.

Visualization

Geometry is best learned in the active mode. Students discover relationships and develop their spatial sense by constructing, drawing, measuring, visualizing, comparing, transforming, and classifying geometric figures. The ability to visualize is particularly important in the study of geometry. Students need to be able to draw images in their minds of how shapes look, and to be able to manipulate these images mentally.

Co-ordinate Geometry

The study of co-ordinate geometry in the Transition Years should include applications (e.g., the graphing of data from experiments and formulas) and also the formal mathematical treatment of co-ordinate geometry on the xy-plane. Students' experiences in graphing and interpreting should include both linear and non-linear relations. With respect to operations involving the xy-plane, by the end of Grade 9, students should have an understanding of the concept of a relation and of the relationship between the form of an equation and the shape of the corresponding graph. They should also have specific skills with respect to linear relations, such as graphing from an equation. Computers and graphics calculators are very powerful aids in studying co-ordinate geometry, and all students should have access to them.

Measurement

The Importance of Measurement

Understanding measurement is essential for all students. In practical terms, students need measurement skills to function in today's world. As well, many teaching models for other topics in mathematics have a measurement base. Measurement concepts are important in other curriculum areas, such as science, environmental studies, and technology. Good instruction in measurement should not only teach skills, but also expand the student's ability to learn and understand mathematics.

Measurement Activities

Students must be actively involved in problem–solving activities if they are to learn about measurement. Understanding of measurement, both attributes and process, must be carefully developed through a variety of activities that allow students to compare, order, and then measure. In order to understand the origin and function of formulas, students need numerous opportunities to explore relationships informally by manipulating objects and models, and to identify generalizations from observed patterns. In the Formative Years, formulas should be proposed by students as the need arises, and should be expressed in their own language.

Attributes of Measurement

Types of measurement, such as length, area, capacity, and volume, need to be encountered on a continual basis in informal but authentic problem–solving settings. Undertaking measurement activities related to other topics in mathematics as well as other curricular areas will help students gain a greater insight into measurement attributes and concepts and develop their understanding of which units are appropriate for particular purposes.

Estimation

Because it is essential for students to learn and apply estimation strategies, estimation should be a natural part of all measurement activities, both in mathematics and in other curricular areas. A variety of estimation strategies should be taught and students should come to understand that the accuracy of the estimate needed will vary according to the circumstances.

Patterning and Algebra

Patterns

Patterns are everywhere in our world. Patterns are the link students need to make connections between mathematics and the world in which they live. Exploring patterns leads students to develop mathematical power and gain an appreciation for the beauty of mathematics.

It is essential for students in the Formative and Transition Years to explore patterns in order to develop understanding of the concept of a variable and of algebraic thinking.

Patterns in Geometry

In geometry, students need opportunities to build structures and study patterns in tiling, wallpaper, quilts, and furniture arrangements. Students can even form human patterns with their bodies in the gym or outdoors.

Patterns in Measurement

Measurement activities provide many opportunities to discover patterns, since to measure means to repeat a standard or non-standard unit with a scale or tool, and formulas are nothing more than the general statement of some observed pattern. Students need an environment that includes many opportunities to measure and to identify patterns.

Patterns in Number and Language

Number patterns are abundant in the student's world. Nature and our modern technological world provide many numerical patterns. The calculator can be used extensively to explore patterns involving repeated digits and skip-counting. Language is another rich source of patterns. Students can examine patterns in sounds, or words, or stories.

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Algebraic Modelling

Algebraic modelling is a powerful method for solving problems. Students begin to learn about algebraic modelling in Grade 9 and continue throughout high school to develop the skills necessary to use it effectively. The foundations of algebraic modelling, however, are established much earlier. Through investigations involving patterns, including stating rules for patterns, students gradually develop the *concept* of a variable. In the Junior Division, students may use words to identify the rule for a pattern. By the Transition Years, students will use symbols to identify pattern rules, and will begin to write formulas that describe simple relationships represented graphically.

Teachers are expected to view algebraic modelling as one strategy amid a wide range of strategies and not leave students feeling that this is memorization work. If a given problem can be solved using other strategies besides algebraic modelling, the teacher should value all reasonable methods.

In Grade 9, formal treatments of algebra begin. Students must learn to solve linear equations and must be given opportunities to acquire the manipulation skills required to do so. Students' experiences with manipulation of algebraic expressions, however, should be limited to simple examples involving the basic operations necessary to solve equations.

Data Management and Probability

Real-World Connections

The revised mathematics program must demonstrate increased attention to data management and probability. Graphing, statistics, and probability are closely linked, and activities involving their use are highly relevant to problem–solving tasks encountered in everyday life. Such activities provide students with the opportunity to think, use, understand, and interpret meanings of number – in other words, to develop number sense.

Activities involving data management and probability provide a meaningful context for the use of computational skills and offer students opportunities for representing, interpreting, and discussing information that applies to other mathematics concepts and skills. These activities help students to develop critical-thinking and problem–solving skills.

Probability

The study of probability in the Formative Years should be kept to an informal/incidental treatment of chance as it relates to games and simple everyday situations. Probability experiments and simulations should be developed in the Transition Years. Theoretical probability calculations should be left to the Specialization Years.

Applications

Graphing, statistics, and probability should not be treated in isolation, but used as tools to apply mathematics concepts and skills in a meaningful context throughout the curriculum. Activities should emphasize collecting and interpreting data, formulating hypotheses, and devising procedures to test them. Activities may range from gathering statistics from projects, concrete graphs, calendars, and weather reports in the early grades to more complex probability experiments in the later grades. At all levels, however, activities must be concrete, manipulative, and based on students' experiences.

Standards of Performance

Problem Solving

The following Sample Performance Indicators apply to students at the end of Grade 3, Grade 6, and Grade 9.

Limited Performance

The student:
– requires motivation from others to engage in problem–solving activities;
– remains on task for only a minimal amount of time;
– will not take risks;
– requires assistance with interpreting the information and identifying the important elements;
– has a limited number of strategies and uses the same one most of the time;
– gives an answer (correct or incorrect) with either no explanation of the solution or an incomplete explanation;
– requires prompting and assistance to determine the reasonableness of the solution;
– uses appropriate technology as directed by the teacher.

Adequate Performance

The student:
– will take risks, when encouraged to do so;
– makes an independent effort and perseveres, when encouraged to do so;
– restates the problem in his or her own words and identifies and organizes the important information, with assistance;
– will construct a new problem using a familiar model and given information, with assistance;
– recognizes problems as they arise during classroom and real-life activities;
– knows a variety of strategies, but needs assistance in choosing the appropriate one;
– uses a variety of communication forms and begins to elaborate upon incomplete explanations, with assistance;
– judges the reasonableness of a solution when directed to do so or when questioned;
– may use appropriate technology independently, while requiring occasional teacher assistance.

Proficient Performance

The student:
– takes risks independently;
– perseveres when solutions are not immediately apparent;

- understands the problem, independently identifies and organizes important information, distinguishes between pertinent and non-pertinent information, and recognizes when additional information is required;
- stays on task for long periods of time;
- knows a variety of strategies and applies them singly or in combination;
- sees the value of encouraging others to participate;
- poses and extends a problem, with assistance;
- identifies potential challenges that require problem–solving strategies arising from relevant classroom and real-life activities;
- communicates solutions effectively in a variety of forms and to a variety of audiences;
- routinely judges the reasonableness of solutions and can estimate an answer to the problem before starting;
- recognizes the value of technology and uses technology as an aid in problem solving.

Superior Performance

The student:
- seeks out challenges that require problem–solving strategies;
- demonstrates a commitment to finding solutions;
- intuitively visualizes and interprets problems;
- poses and extends problems by relating them to a variety of previous experiences;
- compares a variety of strategies and chooses the most effective;
- generates novel strategies to solve a problem;
- uses mathematical language in a precise way to communicate solutions in a variety of forms and to a variety of audiences;
- during the problem–solving process, continually monitors the success and reasonableness of the strategies selected;
- recognizes the value of technology and uses technology effectively and innovatively in problem-solving activities.

End of Grade 3

Number Sense and Numeration

Outcomes From *The Common Curriculum*

Students will:
- understand and apply place value and our numeration system; make concrete, pictorial, and symbolic representations of whole numbers and simple fractions; estimate, read, and write numbers; and estimate answers to simple calculations. (p. 84)
- understand the fundamental operations of addition, subtraction, multiplication, and division of whole numbers and their interrelationships and perform these operations with accuracy. (p. 84)
- use mental calculation and the skills of computational and measurement estimation and commit to memory basic facts, once understood. (p. 85)

More specifically, students will:

- read and print numerals and number words;
- demonstrate understanding of conservation of number in a real-life context;
- compare and order whole numbers;
- create and describe representations of whole numbers using place value;
- create and describe representations of fractions, and of decimals to tenths;
- describe the relationship between counting, grouping, and place value;
- develop an understanding of basic number facts;
- apply computational procedures involving addition and subtraction of whole numbers;
- use a variety of strategies to aid in mental computation;
- use estimation strategies in computations involving whole numbers;
- begin to use the calculator as a mathematical tool;
- construct number meanings through real-world experiences;
- describe how operations can be applied to solving problems.

Standards of Performance

Limited Performance

The student relies on concrete representations and requires assistance in performing basic operations. The student has a good understanding of counting and grouping principles, but demonstrates inconsistencies in applying place value. The student will apply a given strategy in estimation and/or mental mathematics, with assistance.

Sample Performance Indicators

The student:

- uses concrete materials to construct representations of two-digit numbers;
- names the number for a given concrete representation of a two-digit number;
- represents numbers symbolically and in concrete form;
- recalls basic addition and subtraction facts with the assistance of concrete models (i.e., up to $9 + 9$ and $18 - 9$);
- adds and subtracts two- and three-digit numbers without regrouping using concrete materials;
- demonstrates multiplication as repeated addition;
- demonstrates division as repeated subtraction;
- continues to develop strategies for estimation and uses them when directed to do so;
- explores the use of the function keys on a simple four-function calculator;
- identifies some of the important elements in a problem.

Adequate Performance

The student represents, regroups, and renames numbers. The student understands and remembers basic number facts and uses a variety of representations to perform computations. With teacher direction, the student chooses an appropriate method of computation from among estimation, mental mathematics, calculator, or paper-and-pencil procedures.

Sample Performance Indicators

The student:
- reads and prints numerals to 999 and number words to 100;
- counts by 2s, 5s, and 10s up to and down from 100;
- represents two- and three-digit numbers concretely or pictorially;
- represents simple fractions (with denominators of not more than 10) concretely, pictorially, and in written form;
- compares two simple fractions with the same denominator using concrete materials;
- writes an amount of money using decimal notation;
- recalls basic addition and subtraction facts from memory;
- recalls basic multiplication facts up to 5 x 5;
- adds and subtracts two- and three-digit numbers with and without regrouping;
- interprets and records multiplication and division sentences in a variety of ways;
- recognizes when an estimate is appropriate;
- recognizes when a computation could be performed mentally;
- uses a calculator to explore number patterns;
- identifies and uses the basic operation required to solve a one-step problem.

Proficient Performance

The student describes how counting, grouping, and place value are related to one another. The student sees relationships among the operations and uses a variety of methods in performing them. The student uses estimation and mental mathematics strategies and the calculator as a tool where appropriate.

Sample Performance Indicators

The student:
- names and renames quantities to 999 (e.g., 52 as 52 ones, 5 tens and 2 ones, fifty-two, etc.);
- uses place value to order three-digit numbers;
- represents equivalent fractions using concrete materials;
- represents decimal tenths using concrete materials;
- adds and subtracts amounts of money using decimal notation;
- recalls basic division facts up to 25 ÷ 5;
- adds and subtracts two- and three-digit numbers with and without regrouping using a variety of methods (e.g., adding on, using patterns, etc.);
- solves multiplication and division sentences using a variety of methods and materials;
- uses estimation strategies to solve problems and to check if results are reasonable;
- explores, invents, and shares a variety of mental mathematics strategies;
- recognizes when the use of a calculator is appropriate;
- uses a calculator in a variety of ways (e.g., to perform operations, examine number relationships, solve problems, etc.);
- solves problems involving more than one operation;
- creates problems from given information.

Superior Performance

The student has an intuitive sense of number and a sound understanding of our numeration system. The student creates and applies a variety of strategies for estimation and mental mathematics. The student uses these skills in combination to make judgements about the reasonableness of computational results and of proposed solutions.

Sample Performance Indicators

The student:
- applies place-value concepts to numbers beyond 999;
- illustrates improper fractions using concrete materials;
- identifies decimal parts of a whole;
- compares and orders decimal numbers;
- adds and subtracts numbers with more than three digits;
- applies relationships among the operations ($+$, $-$, $\times$, $\div$) where appropriate;
- creates and applies a variety of strategies for mental mathematics and estimation;
- recognizes the capabilities and limitations of a calculator;
- creates and solves problems that demonstrate the basic operations.

Geometry and Spatial Sense

Outcomes From *The Common Curriculum*

Students will:
- see relationships among objects, images, and systems with respect to size, shape, colour, strength, and texture. (p. 78)
- demonstrate understanding of the spatial relationships in their own environment, the properties of the objects in it, their interrelationships, and the effects of motion geometry. (p. 84)
- demonstrate understanding of the attributes of two-dimensional figures and three-dimensional shapes. (p. 85)

More specifically, students will:
- identify and sort two-dimensional and three-dimensional shapes;
- construct two-dimensional and three-dimensional frames using a variety of concrete materials;
- construct a variety of three-dimensional structures;
- generate geometric patterns in two and three dimensions;
- investigate symmetry;
- explain movements that take an object from one position to another;
- solve problems in geometric contexts.

Standards of Performance

Limited Performance

The student uses language in a general way to express ideas about geometric objects and their relationships. The student shows a high degree of dependence on the presence and use of concrete materials to represent, demonstrate, or express geometric ideas. With support, the student

can produce written records of work. The student begins to notice and describe ways of positioning objects in space.

Sample Performance Indicators

The student:
– sorts a collection of items using a given criterion (e.g., number of sides, shape);
– builds a three-dimensional structure using a variety of materials and names the solids used;
– describes and continues two-dimensional patterns and names basic shapes used;
– uses his or her own language to describe positions of objects in space;
– recognizes problems encountered in geometric activities.

Adequate Performance

The student begins to use correct geometric terminology to describe specific properties and relationships of two- and three-dimensional shapes. The student independently uses a given framework to make written records of geometric activities while manipulating or observing shapes in the environment. The student begins to notice and describe motions of objects in space.

Sample Performance Indicators

The student:
– sorts and re-sorts a collection of items using his or her own stated criterion;
– describes similarities and differences in student-built three-dimensional structures (e.g., number of faces, edges, vertices);
– creates two- and three-dimensional patterns using at least three shapes (e.g., cylinder, cube, triangular prism);
– identifies slides, flips, and turns as motions of objects;
– uses previous experience to assist in solving geometric problems.

Proficient Performance

The student works independently with materials to create models and demonstrate understanding about the characteristics of two- and three-dimensional shapes and about how objects move in space. The student explains geometric ideas clearly and uses a variety of methods to record them.

Sample Performance Indicators

The student:
– creates groups of objects using more than one criterion and records the results;
– builds a three-dimensional structure and describes how symmetry, pattern, strength, and stability are reflected in it;
– creates two- and three-dimensional patterns using more than one criterion and records the results;
– demonstrates slides, flips, and turns through movement of objects and his or her own body;
– suggests possible ways to solve geometric problems.

Superior Performance

The student uses mathematical language and concepts in creative ways to generate and examine geometric situations.

Sample Performance Indicators

The student:
- generates and records intersecting sets (e.g., Venn diagrams, matrix);
- uses knowledge of the characteristics of three-dimensional solids to solve a problem (e.g., constructing a bridge to support a given weight);
- creates complex geometric patterns involving several attributes;
- creates designs using combinations of slides, flips, and turns;
- describes or demonstrates explorations, decisions, and solutions to geometric problems.

Measurement

Outcomes From *The Common Curriculum*

Students will:
- understand the concepts of temperature, time, distance, speed, length, perimeter, area, capacity/volume, mass, and money and, where applicable, use standard (SI metric) and non-standard units to estimate, measure, and compare quantities. (p. 85)

More specifically, students will:
- estimate to make predictions of measurements;
- measure length, mass, capacity/volume, area, and time using non-standard units;
- measure length, mass, capacity/volume, time, and temperature using standard units;
- select an appropriate unit to use for estimation and measurement;
- record measurement results in a variety of ways;
- use measurement concepts to solve everyday problems;
- identify, compare, and use coins and paper money, recording the amounts involved.

Standards of Performance

Limited Performance

The student understands how to measure using non-standard units, and identifies which tools are appropriate for a given measurement task. With support, the student records results of activities and begins to refine the accuracy of the language used to describe differences.

Sample Performance Indicators

The student:
- compares three objects using broad measurement terms (e.g., longest, heaviest);
- measures a given object using three or more non-standard units;
- recognizes which measurement tool is used for length, mass, temperature, time, capacity;
- reads time to the hour and half-hour (analog and digital);
- records results of a measurement task with direction.

Adequate Performance

The student has a good understanding of the purpose of estimation in measurement and begins to use standard units. In oral and written work, the student shows understanding of the need to communicate differences in measurement accurately.

Sample Performance Indicators

The student:
- distinguishes between making a "wild" guess and making an estimate based on past experience;
- explains the need for a common unit of measurement;
- compares the mass of various objects to one kilogram (capacity to one litre, etc.);
- measures perimeter using non-standard and standard units;
- recognizes the need for larger and smaller measurement units such as metre and centimetre;
- records results of a measurement task independently.

Proficient Performance

The student uses estimation effectively to predict measurement outcomes. The student accurately uses the appropriate standard unit of measurement for a given task. The student independently applies alternative ways of recording results of measurement activities.

Sample Performance Indicators

The student:
- uses previous estimates to make more accurate predictions of measurements within a given task;
- estimates and measures length in metres (centimetres); mass in kilograms (grams); capacity in litres (millilitres); time (five-minute intervals); temperature (degrees Celsius), etc.;
- judges when to use larger or smaller measurement units (e.g., metres or centimetres);
- measures and compares areas using non-standard units;
- creates graphs, diagrams, etc., to report on a measurement activity.

Superior Performance

The student has internalized measurement concepts and applies them with a high degree of accuracy to solve everyday problems.

Sample Performance Indicators

The student:
- develops a set of personal reference points upon which to base measurement estimates;
- demonstrates the relationship between a metre and a centimetre, etc.;
- creates measurement problems choosing appropriate units.

Patterning and Algebra

Outcomes From *The Common Curriculum*

Students will:
- describe, extend, generalize from, and create patterns in order to build mathematical models and solve everyday problems. (p. 78)
- identify, create, and generalize from patterns and series. (p. 85)
- use calculators and computers to explore simple patterns and relationships and solve problems relating to familiar objects and settings. (p. 95)

More specifically, students will:
- explore a wide variety of patterns in geometry, measurement, and number;
- use patterns to represent and solve problems;
- represent and describe mathematical relationships.

Standards of Performance

Limited Performance

The student recognizes given patterns in his or her everyday environment. The student uses a variety of mathematical symbols to represent number relationships.

Sample Performance Indicators

The student:
- recognizes and describes similarities and differences in shape, size, and quantity using concrete materials and real-world examples;
- recognizes, describes, and extends a given pattern involving concrete or pictorial representations;
- recognizes signs (e.g., $+$, $-$, $>$, $<$, $=$, $\neq$) and uses them to denote mathematical relationships.

Adequate Performance

The student independently creates and describes a variety of patterns and becomes aware of how patterns can assist with problem solving. The student represents relationships among numbers through the use of conventional signs of operations.

Sample Performance Indicators

The student:
- recognizes, describes, extends, and creates a wide variety of repeating patterns (e.g., ABC, ABC, ...) using objects and numbers;
- shows how patterns can be used to solve problems that are designed to reflect repetition;
- constructs number-fact families for addition and subtraction (e.g., $2 + 5 = 7$, $5 + 2 = 7$, $7 - 2 = 5$, $7 - 5 = 2$);
- recognizes multiplication ($\times$) and division ($\div$) signs and uses them to denote mathematical relationships;

- recognizes the patterns of multiplication as repeated addition, and of division as repeated subtraction;
- explores number patterns using a calculator.

Proficient Performance

The student understands a variety of pattern structures and independently uses these to solve problems. The student correctly interprets the relationships created among groups of numbers when conventional mathematical signs are applied. The student begins to use this knowledge to predict missing terms in an equation.

Sample Performance Indicators

The student:
- uses concrete materials to copy, extend, and create geometric patterns;
- recognizes, describes, extends, and creates a wide variety of growing patterns (e.g., A, AB, ABC, ... or ABC, ABBC, ABBBC, ...) using objects and numbers;
- creates numerical patterns by skip-counting, adding on, or subtracting and solves problems using these patterns;
- applies knowledge of number-fact families to determine the missing term in a simple equation (e.g., $2 + \underline{?} = 7$);
- demonstrates the relationships between addition and multiplication and between subtraction and division in a given context;
- identifies counting patterns on a hundreds chart.

Superior Performance

The student has a highly developed sense of pattern and uses this knowledge intuitively as a strategy for designing and solving problems.

Sample Performance Indicators

The student:
- develops repeated and growing number patterns involving increasing complexity;
- uses patterns intuitively as a strategy for solving problems;
- recognizes and applies the relationship between multiplication and division when solving problems.

Data Management and Probability

Outcomes From *The Common Curriculum*

Students will:
- understand a variety of ways in which information can be organized, analysed, interpreted, and stored. (p. 75)
- begin to understand and use a variety of graphic symbols and conventions employed in different types of graphs, charts, models, and diagrams, with reference to reading and making maps (distance, scale, direction, location), telling time, observing, collecting, and recording data in field work, etc. (p. 75)

- use a variety of investigative strategies to gather, analyse, and interpret information drawn from their immediate natural and human-made environments and communicate the information clearly. (p. 94)
- demonstrate a beginning understanding of the concepts of chance and probability through informal investigations involving data collection and interpretation. (p. 94)

More specifically, students will:
– collect and organize information related to familiar situations;
– display and record data;
– interpret and compare displayed information;
– predict the probability that a particular event will occur;
– use concepts of chance to solve problems;
– apply data-management skills throughout the mathematics curriculum and across other subject areas.

Standards of Performance

Limited Performance

The student uses data-management skills with support and begins to apply the vocabulary of chance to describe everyday experiences.

Sample Performance Indicators

The student:
– collects and organizes materials and/or information according to specific criteria;
– completes partially constructed tables or graphs;
– answers specific questions in order to make comparisons about displayed data;
– uses appropriate vocabulary related to concepts of chance (e.g., often, sometimes, never, impossible, likely, probably);
– makes reasonable predictions about situations based on personal experiences.

Adequate Performance

The student begins to use data-management skills independently to demonstrate relationships. The student begins to make reasoned predictions using data.

Sample Performance Indicators

The student:
– decides collaboratively what information is to be collected (i.e., learns how to formulate an appropriate question);
– gathers and organizes data by conducting a survey or interview based on predetermined criteria;
– constructs pictographs and bar graphs using a 1:1 scale;
– interprets information from displayed data and lists results;
– uses concepts of chance and data from common experience to make reasonable predictions.

Proficient Performance

The student uses data-management skills independently to draw conclusions. Through informal investigations, the student begins to link expectations that particular events will occur with notions of chance.

Sample Performance Indicators

The student:
- creates a relevant survey question;
- gathers and organizes data by conducting a survey or interview based on several criteria;
- constructs pictographs and bar graphs using multiple scales (2, 5, 10);
- analyses information from displayed data and draws conclusions;
- predicts results of simple probability experiments (e.g., rolling a number cube, spinning a spinner) and/or games of chance, and tests the predictions by trying the experiments.

Superior Performance

The student applies data-management skills in cross-curricular contexts. The student regularly uses concepts of chance to predict outcomes of events.

Sample Performance Indicators

The student:
- designs and conducts a survey or interview;
- constructs different types of graphs and compares their suitability in various situations;
- discovers and verifies patterns in graphs;
- predicts the probability that a particular event will occur and then provides reasonable data to support the prediction;
- designs games or experiments using concepts of chance.

End of Grade 6

Number Sense and Numeration

Outcomes From *The Common Curriculum*

Students will:
- understand the relationships among events, time, distances, and speed/velocity. (p. 79)
- identify and understand whole-number place values and decimal place values; estimate, read, and write numbers; and estimate answers to calculations. (p. 84)
- select and use an appropriate method of computation in solving a problem that involves computations using whole numbers, fractions, percentages, and decimals. (p. 84)
- value and use mental calculations and the skills of computational and measurement estimation through independent use; and commit to memory basic information such as the multiplication table to 12 x 12. (p. 85)
- use calculators and computers to explore patterns and relationships and solve problems relating to a variety of settings, from the familiar to the global. (p. 95)

More specifically, students will:
- demonstrate number sense in operating with whole numbers, decimals, and fractions;
- use mental mathematics, estimation, calculators, and pencil-and-paper procedures involving whole numbers, decimals, and fractions;
- in a given situation, choose the most appropriate calculation method from among mental mathematics, estimation, calculator, or pencil-and-paper procedures;
- demonstrate an understanding of rate and ratio based on experiences in other subjects (e.g., scale diagrams and map skills in Social Studies, gear ratios in Technology, rates in Science and Environmental Studies);
- demonstrate a beginning understanding of per cent in relation to concrete models and pictorial representations;
- solve a wide variety of single and multi-step problems drawn from school and everyday situations.

Standards of Performance

Limited Performance

The student requires the use of models, concrete materials, and teacher assistance in understanding place value and carrying out computational procedures. The student demonstrates limited success in computations involving whole numbers, decimals, and fractions.

Sample Performance Indicators

The student:
- in solving problems, requires assistance in identifying the arithmetical operation involved (addition, subtraction, multiplication, division);
- demonstrates limited understanding of the effect of operations on whole numbers;
- demonstrates inconsistent understanding of place value and has difficulty in applying it to reading and writing numbers and in estimation and calculation;
- demonstrates partial recall of basic number facts;
- demonstrates little understanding of pencil-and-paper procedures involving whole numbers and decimals;
- with the assistance of prompting and concrete materials, demonstrates partial understanding of basic concepts involving fractions such as representation, magnitude, order, and equivalence;
- with assistance, demonstrates understanding of basic procedures in mental calculation and estimation and applies them with limited success;
- uses a calculator to support learning and application of basic facts, pencil-and-paper procedures, and skills in mental mathematics.

Adequate Performance

The student demonstrates understanding of the properties and operations related to whole numbers, decimals, and fractions. With prompting, the student readily recognizes and corrects errors. The student continues to develop number sense.

Sample Performance Indicators

The student:

- in a one-step arithmetic problem, identifies the appropriate operation to use;
- understands basic laws of operations on numbers and applies them when prompted (e.g., $2 + 3 = 3 + 2$; $[6 \times 4] \times 5 = 6 \times [4 \times 5]$);
- demonstrates understanding of the effects of operations on whole numbers (e.g., $65 \div 5$ produces a value less than 65, but 65×5 produces a value greater than 65);
- demonstrates understanding of place value, but may require prompting in applying it consistently in reading and writing numbers and in estimation and calculation;
- recalls basic number facts with some hesitation;
- demonstrates inconsistency in doing pencil-and-paper procedures involving whole numbers and decimals;
- with the assistance of concrete models, demonstrates an understanding of basic concepts involving fractions, such as representation, magnitude, order, and equivalence;
- with the assistance of concrete materials, adds and subtracts fractions with simple denominators and multiplies a fraction by a whole number;
- in school and everyday situations, applies basic skills of mental calculation with inconsistent accuracy;
- in school and everyday situations, applies estimation by rounding with varying degrees of success;
- uses a calculator effectively in applications and problem solving throughout the curriculum and in everyday life;
- uses a calculator to investigate concepts in mathematics;
- chooses the most appropriate calculation method from among mental mathematics, estimation, calculator, or paper-and-pencil procedures.

Proficient Performance

The student uses a variety of calculation modes effectively to solve problems drawn from across the curriculum and from everyday life. The student exhibits good number sense.

Sample Performance Indicators

The student:

- in a two-step arithmetic problem, identifies the appropriate operation to be used;
- applies basic laws of operations on numbers in calculation and estimation (e.g., $130 + 56 + 70$ is simplified mentally as $[130 + 70] + 56$; $5 \times 9 \times 2$ is simplified mentally as $[5 \times 2] \times 9$);
- demonstrates understanding of the effects of operations on whole numbers and decimals (e.g., $65 \div 0.5$ produces a number larger than 65);
- demonstrates solid understanding of place value and applies it effectively in reading and writing numbers and in estimation and calculation;
- demonstrates quick recall of basic arithmetic facts;
- demonstrates proficiency in pencil-and-paper procedures involving whole numbers and decimals, to reasonable levels of difficulty (e.g., can add up to three four-digit numbers, subtract two four-digit numbers, multiply a two-digit number by a two-digit number, divide by a one-digit divisor);

- demonstrates a confident understanding of basic concepts involving fractions, such as representation, magnitude, order, and equivalence;
- proficiently adds and subtracts fractions with simple denominators, and multiplies a fraction by a whole number;
- in school and everyday situations, effectively applies basic skills of mental calculation (e.g., can add, subtract, multiply, and divide numbers ending in zeros);
- in school and everyday situations, effectively applies estimation by rounding.

Superior Performance

The student has an intuitive sense of number and applies it effectively in solving problems involving calculation and numerical reasoning.

Sample Performance Indicators

The student:
- in multi-step arithmetic problems, identifies the appropriate operations to be used;
- demonstrates facility with and intuition in the use of whole numbers, decimals, and fractions;
- uses a calculator creatively to carry out independent explorations of concepts and relationships in mathematics;
- identifies and applies a variety of estimation strategies;
- routinely and appropriately applies mental mathematics and estimation as an integral part of calculation.

Geometry and Spatial Sense

Outcomes From *The Common Curriculum*

Students will:
- use motion geometry to verify the basic properties of figures and to recognize and create patterns and designs. (p. 84)
- describe, model, draw, construct, classify, and compare two-dimensional and three-dimensional figures, explain their properties using accurate terminology, and use Cartesian co-ordinates systems to describe the locations of points and figures and represent relationships defined by tables and graphs. (p. 85)

More specifically, students will:
- measure and construct angles using a protractor;
- classify angles by size;
- identify and construct intersecting, parallel, and perpendicular lines;
- identify two-dimensional figures and three-dimensional objects;
- describe and classify two-dimensional figures and three-dimensional objects;
- recognize and describe congruent figures;
- identify symmetry in two-dimensional figures and three-dimensional objects;
- use concrete materials in hands-on activities to apply slides, flips, and turns;
- pose and solve problems in a geometric context.

Standards of Performance

Limited Performance

The student uses the language of geometry loosely to describe the world. The student is beginning to make discriminations among members of classes of figures and objects but is not able to articulate the comparisons clearly. The student demonstrates an emerging spatial understanding of the effect of motion in geometry.

Sample Performance Indicators

The student:
– selects a protractor when asked to measure or construct angles;
– estimates angle size within a reasonable range;
– recognizes and names two-dimensional figures and three-dimensional objects in a variety of contexts;
– describes the properties of two-dimensional figures and three-dimensional objects using appropriate language;
– constructs a three-dimensional object from a drawing using appropriate materials;
– recognizes similarities and differences among two-dimensional figures;
– recognizes similarities and differences among three-dimensional objects;
– recognizes mirror symmetry in two-dimensional figures and three-dimensional objects;
– identifies congruent figures by matching;
– identifies lines of symmetry in two-dimensional figures;
– applies slides, flips, and turns to concrete materials to make designs;
– given a shape and its image under a single motion, identifies the motion.

Adequate Performance

The student uses the language of geometry appropriately to describe the world and makes distinctions in classifying figures and objects. The student demonstrates an understanding of the effect of motion in geometry.

Sample Performance Indicators

The student:
– uses a protractor to measure and construct angles with reasonable accuracy;
– classifies angles;
– classifies two-dimensional figures according to angle and side properties;
– classifies three-dimensional objects according to face, edge, and vertex properties;
– constructs three-dimensional objects from given descriptions;
– proves that a two-dimensional figure has mirror symmetry by identifying the mirror line;
– creates a two-dimensional figure or three-dimensional object that has mirror symmetry;
– identifies congruent figures by measuring side length and angle size;
– recognizes slides, flips, and turns in patterns and designs in the environment;
– solves problems in geometric contexts that involve the direct application of learned concepts and skills.

Proficient Performance

The student uses the language of geometry effectively to describe shapes in the environment and to discuss geometric ideas. The student understands the effects of motion geometry and creates related patterns and designs.

Sample Performance Indicators

The student:
– uses slides, flips, and turns to generate patterns and designs;
– visualizes and describes the effects of slides, flips, and turns, singly and in combinations;
– uses computer programs such as Logo to explore and apply slides, flips, and turns;
– solves problems in geometric contexts that involve seeing relationships between or among concepts and/or skills;
– generates alternative solutions to problems in geometric contexts;
– recognizes and describes the occurrence or application of geometric properties and principles in the real world and in other subjects.

Superior Performance

The student has internalized the language of geometry and independently generates novel applications of learned geometric concepts.

Sample Performance Indicators

The student:
– poses and solves a wide variety of geometry problems that involve relating geometry to other strands of mathematics and to other subjects;
– draws inferences and makes logical deductions in solving geometry problems.

Measurement

Outcomes From *The Common Curriculum*

Students will:
• estimate, measure, calculate, and record temperature, length, perimeter, area, capacity/volume, mass, money, time, distance, and velocity, using appropriate units. (p. 85)

More specifically, students will:
– understand the difference between estimation and precise measurement and use the strategies appropriately in everyday situations;
– demonstrate understanding of the concepts of perimeter, area, capacity, volume, and mass;
– estimate and measure length, perimeter, area, capacity, volume, and mass using standard units;
– estimate and measure in standard units the perimeter and area of irregular figures and the capacity, volume, and mass of irregular shapes;
– solve problems drawn from everyday life involving estimation and measurement of length, perimeter, area, capacity, volume, and mass;
– identify formulas for the perimeter and area of a rectangle and square and solve problems involving them;

- identify the relationship between the twelve-hour clock and the twenty-four-hour clock and read both analog and digital time devices;
- estimate rates involving time, such as speed of walking or running;
- estimate and measure temperature in standard units;
- develop personal referents for measures involving length, perimeter, area, capacity, volume, mass, temperature, and time through hands-on measurement and estimation activities.

Standards of Performance

Limited Performance

The student uses appropriate instruments to measure length, capacity, and mass in standard units. The student demonstrates a rudimentary understanding of the concepts and applications of perimeter, area, and volume. The student has limited estimation skills.

Sample Performance Indicators

The student:
- chooses the appropriate unit for a given task and carries out the measurement;
- determines perimeter, area, and volume, with assistance;
- recognizes that measurement is important in other areas of the curriculum.

Adequate Performance

The student measures with reasonable accuracy using standard units. The student demonstrates an emerging understanding of the concepts of perimeter, area, and volume, and the relationships among them. The student solves problems involving a direct application of learned measurement skills.

Sample Performance Indicators

The student:
- uses models or concrete materials to illustrate the concepts of perimeter, area, and volume;
- begins to use personal referents for estimating measurements;
- demonstrates the relationship between metres and centimetres, litres and millilitres, grams and kilograms;
- uses measurement to solve everyday problems.

Proficient Performance

The student applies measurement concepts and skills with understanding to solve a wide variety of practical problems. The student sees the value of estimation and has begun to apply a set of personal referents in making reasonable estimates.

Sample Performance Indicators

The student:
- demonstrates understanding of the concepts of perimeter and area and applies formulas with understanding;
- distinguishes between capacity and volume;

- measures with precision appropriate to the instrument and the context;
- devises reasonable methods for measuring the perimeter, area, volume, or capacity of irregular figures or objects;
- understands the prefixes used in the metric system and can make reasonable conversions (e.g., millilitre to litre; milligram to gram; metre to kilometre);
- recognizes ways in which measurement is important in other areas of the curriculum or everyday life and uses measurement skills to solve problems;
- uses mathematical language effectively to discuss ideas, techniques, and findings involving measurement;
- applies a set of personal referents to estimate length, area, volume, capacity, mass, time, and temperature.

Superior Performance

The student estimates effectively and routinely applies an understanding of measurement in all areas of the curriculum and in everyday life. The student demonstrates insight into the relationships between different types of measures.

Sample Performance Indicators

The student:
- uses an effective set of personal referents for estimation;
- uses independent reasoning to identify formulas for perimeter and area;
- creates and solves problems involving relationships between and among different aspects of measurement (e.g., A rectangle has an area of 24 cm^2. What possible rectangles could be constructed on a 1-cm grid and what are their perimeters?).

Patterning and Algebra

Outcomes From *The Common Curriculum*

Students will:
- describe, extend, generalize from, and create patterns and explore the relationships among them. (p. 78)
- identify patterns and functions, describe them in a variety of ways, and explain the relationships among them. (p. 85)

More specifically, students will:
- identify patterns in calculation and mental computation;
- identify, extend, and create number patterns involving whole numbers and decimals;
- identify, extend, and create patterns in geometry;
- describe patterns in words but not necessarily symbolically;
- identify formulas as generalizations about observed patterns;
- recognize and identify patterns in material from other subject areas and in the world around them;
- solve problems by applying a patterning strategy.

Standards of Performance

Limited Performance

The student uses concrete materials to assist in recognizing and creating patterns. The student demonstrates limited ability to describe patterns.

Sample Performance Indicators

The student:
- uses concrete materials to create geometric patterns;
- identifies one or more transformations applied to a geometric shape;
- identifies patterns in a hundreds chart;
- creates number patterns involving whole numbers;
- recognizes patterns but demonstrates limited ability in describing them in his or her own words.

Adequate Performance

Through exposure to a wide variety of patterns, the student begins to recognize a variety of rules for making patterns, and can apply them with support. The student demonstrates an emerging ability to identify and describe patterns without reference to concrete materials.

Sample Performance Indicators

The student:
- identifies, extends, and creates geometric patterns beyond the use of concrete materials;
- identifies patterns in multiplication tables;
- creates number patterns involving whole numbers and decimals;
- identifies and extends number patterns involving whole numbers and decimals;
- creates geometric patterns from numeric patterns (e.g., can use base 10 or other materials to represent 0.25, 0.50, 0.75, ...);
- recognizes patterns in the metric system;
- uses his or her own words to describe a pattern.

Proficient Performance

Through exposure to a wide variety of patterns, the student develops a repertoire of strategies and rules for patterning. The student demonstrates increasing intuition in the ability to identify patterns and describes them effectively using mathematical language. The student begins to recognize the power of patterning in mathematics.

Sample Performance Indicators

The student:
- identifies, extends, and creates numeric patterns involving whole numbers and decimals;
- relates geometric patterns to numeric patterns;
- identifies formulas for perimeter and area of a rectangle as generalizations of observed patterns;

- uses appropriate mathematical language to describe patterns;
- records conclusions involving patterns using mathematical formats such as tables and graphs;
- demonstrates an emerging ability to recognize mathematical patterns in material from other subject areas and in the surrounding world.

Superior Performance

By stating generalizations to patterns and sequences symbolically, the student demonstrates an emerging understanding of the concept of a variable.

Sample Performance Indicators

The student:
- represents patterns symbolically;
- recognizes mathematical patterns in material from other subject areas and in the surrounding world.

Data Management and Probability

Outcomes From *The Common Curriculum*

Students will:
- use investigation skills to gather, analyse, interpret, and evaluate information, communicate the information clearly, and apply it appropriately. (p. 94)
- understand and explain the concepts of chance and probability through investigations involving the collection, organization, interpretation, and application of statistical information. (p. 94)

More specifically, students will:
- carry out surveys to gather data for constructing graphs;
- read and interpret information from bar graphs, pictographs, broken-line graphs, and circle graphs;
- construct bar graphs, pictographs, and broken-line graphs;
- read and discuss examples of graphs found in print and non-print media;
- apply graphing skills in other areas of the curriculum;
- develop an intuitive sense of chance/probability by playing and constructing a variety of games (e.g., in number cubes/card games);
- discuss chance/probability as it occurs in everyday situations.

Standards of Performance

Limited Performance

The student collects, organizes, and graphs data with assistance. The student can read data from tables and graphs. The student recognizes elements of chance in everyday occurrences.

Sample Performance Indicators

The student:
– with assistance, collects and organizes data for a given question;
– reads and constructs bar graphs and broken-line graphs in which the scale represents a one-to-one correspondence (scale in units);
– reads and constructs pictographs in which the scale represents a many-to-one correspondence (e.g., scale in 2s or 10s or 100s);
– views success in board/card games as the result of luck rather than of strategy based on knowledge of probability;
– in everyday situations, recognizes certainty and impossibility and that some events are more or less likely to occur.

Adequate Performance

The student consistently collects and organizes data systematically and represents them in an appropriate way. The student makes basic interpretations of data to answer questions. The student begins to recognize that there is a relationship between external factors and the role of chance in everyday occurrences.

Sample Performance Indicators

The student:
– carries out a survey to collect data;
– designs methods to organize collected data;
– reads, interprets, and constructs graphs in which the scale represents a many-to-one correspondence (e.g., scale in 10s or 100s);
– recognizes appropriate terminology related to methods and findings in surveys and graphs;
– begins to recognize that some events occur more frequently than others (e.g., in board/card games);
– understands the meaning of commonly used statements relating to chance (e.g., a fifty-fifty chance, odds of three to one, the prediction possibilities of a batting average);
– in everyday situations, predicts the possible outcomes of a situation and explains his or her reasoning (e.g., makes a prediction about the best time to go to a restaurant).

Proficient Performance

The student collects, organizes, and displays data efficiently. The student interprets data represented in graphic form. The student has an emerging sense of the role of chance in everyday situations and considers external factors that affect outcomes.

Sample Performance Indicators

The student:
– makes a prediction about the response to a survey question before carrying out the survey;
– independently carries out a survey and organizes the data gathered;
– compares information about the same question gathered from different sources;
– uses the language of surveys and graphs effectively to discuss methods and findings;

- writes a description of the results of a survey;
- recognizes the use and prevalence of data management in society;
- makes decisions based on observation of frequency of occurrence of events (e.g., in number cubes/board/card games);
- recognizes and discusses realistic chances of winning in games of chance, such as lotteries, raffles, bingo.

Superior Performance

The student demonstrates a unique perspective on data-management activities in real life. The student makes sophisticated comparisons of data, including the consideration of data beyond the direct scope of the question. The student has an intuitive sense of chance.

Sample Performance Indicators

The student:
- selects the appropriate graph for a given set of data;
- critically evaluates the process used in a data-management activity (e.g., questions the effect on survey results of limited sample size);
- demonstrates an intuitive sense of chance as it relates to games of chance and everyday situations.

End of Grade 9

Number Sense and Numeration

Outcomes From *The Common Curriculum*

Students will:
- understand the relationships among events, time, distances, and velocity. (p. 79)
- use computational estimation and mental calculation to arrive at approximate value, and judge the validity or reasonableness of answers in a variety of problem–solving contexts. (p. 84)
- demonstrate in context, with and without the use of a calculator and/or computer, a thorough knowledge of arithmetic facts and accurate computational and mental calculation skills involving whole numbers, decimals, fractions, per cent, integers, ratio and rate, powers, and square root. (p. 84)
- value and use mental calculations and the skills of computational and measurement estimation, and be able to apply those skills to new problem–solving tasks. (p. 85)

More specifically, students will:
- use estimation and mental calculation to make decisions about situations that are common in everyday life;
- use a four-function calculator and a scientific calculator as tools for calculation and use computer spreadsheets for more complex tasks;
- use a calculator to learn about and investigate mathematical concepts;
- judge the reasonableness and accuracy of numeric information from a variety of media sources;
- assess whether an estimate or an exact answer is required in a problem–solving activity;

- develop an understanding of integers, ratio and rate, per cent, fractions, powers, and roots and the uses of these numeric tools in real-life situations;
- use appropriate methods of computation involving whole numbers, decimals, per cent, integers, ratio and rate, powers, and square root in applications, problem solving, and problem posing.

Standards of Performance

Limited Performance

The student constructs number meanings from real-world experiences and uses concrete materials to develop understandings involving counting, grouping, and place-value concepts. The student realizes that numbers are used in the world in a variety of ways. The student relies on a calculator to do most of the calculations encountered in an application.

Sample Performance Indicators

The student:
- requires assistance to determine the type of numerical calculation called for in an application;
- estimates in a mechanical way with little or no intuitiveness;
- requires concrete representations for operations with integers, decimals, and percentages;
- requires assistance to determine whether an estimate or an exact answer is appropriate in a given application;
- uses the calculator to do individual operations and record each answer individually;
- solves one-step problems with teacher assistance.

Adequate Performance

The student understands properties and operations involving number, but may refer to memorized rules. The student uses mental mathematics and estimation strategies to support the effective use of a calculator.

Sample Performance Indicators

The student:
- determines the type of numerical calculation called for in an application;
- uses estimation when asked to determine whether an answer appears to be correct;
- determines whether an estimate or an exact answer is appropriate in a given application;
- strings together more than one operation on a calculator and begins to understand and investigate the power of the calculator;
- solves multi-step problems and poses additional problems, with teacher assistance.

Proficient Performance

The student works independently and co-operatively, seeking assistance when appropriate. The student has a thorough knowledge of how to apply number in a multiplicity of contexts and usually does so with ease and accuracy.

The student:
- usually comes up with the correct answer using an appropriate method of calculation;
- estimates the answer to a calculation called for in an application before determining the exact answer and thus is able to recognize his or her own errors;
- with assistance, uses a spreadsheet to do the required calculation in an application where it is helpful;
- uses the calculator efficiently;
- uses the calculator to investigate and understand number relationships and patterns;
- poses and solves a variety of problems.

Superior Performance

The student has a keen sense of number and easily applies this understanding to problems both in and outside the school setting.

Sample Performance Indicators

The student:
- estimates an answer in a number of different ways and will choose the most appropriate one in each situation;
- chooses the appropriate method of calculation (including spreadsheets), given a problem involving a numerical calculation;
- uses a calculator efficiently and to its full potential for calculation and investigation;
- uses a variety of strategies to solve and pose multi-step problems.

Geometry and Spatial Sense

Outcomes From *The Common Curriculum*

Students will:
- demonstrate understanding of spatial and geometric relationships by investigating two-dimensional, three-dimensional, and transformational geometry. (p. 84)

More specifically, students will:
- construct and measure an angle with a protractor with reasonable accuracy;
- construct or draw two-dimensional shapes such as triangles, quadrilaterals, and regular polygons, given oral or written instructions;
- classify, identify, name, and locate two-dimensional shapes (e.g., angles, triangles, quadrilaterals) in their environment;
- carry out basic constructions using a variety of tools and techniques, such as paper folding, MIRA, computer, protractor, and ruler and compass;
- identify congruent shapes;
- identify and describe the properties of two-dimensional shapes such as congruent parts, line symmetry, rotational symmetry, angle properties;
- identify the angle properties of intersecting, parallel, and perpendicular lines;
- solve numeric problems using the properties of two-dimensional shapes and the angle properties of intersecting, parallel, and perpendicular lines;

- identify the properties of translations, reflections, rotations, and dilatations; draw the images of two-dimensional shapes under each transformation; given a figure and its image, state the transformation that has produced the image;
- analyse designs and use a variety of techniques to create designs involving geometric shapes;
- identify figures that will tile the plane and create tessellations of the plane identifying associated symmetries and patterns;
- create similar figures and scale drawings; identify the properties of similar figures and use these properties to interpret scale drawings (e.g., maps) and scale models;
- plot the images of transformations in the Cartesian plane;
- sketch three-dimensional figures on dot paper and construct three-dimensional figures, using cubes from their top, side, and front views.

Standards of Performance

Limited Performance

The student is very dependent on teacher direction. The student relies on concrete materials to assist in the visualization of geometric ideas. The student exhibits little understanding of geometric concepts and has limited retention of information about them. The student has a strong need for aids such as diagrams and posters showing geometric definitions and requires direction in their use. The student assumes that mathematics is done in isolation. The student has limited ability to articulate results or explain procedures and solutions to other students.

Sample Performance Indicators

The student:
- completes construction tasks in two or three dimensions, given the appropriate tools or materials and detailed direction one step at a time;
- identifies geometric figures in the environment but has difficulty describing them;
- solves one-step problems with direction;
- uses available geometry-related programs on the computer with assistance.

Adequate Performance

The student is frequently dependent on the teacher or other students for assistance. The student frequently needs to use concrete materials to establish or clarify understanding of geometric concepts. The student understands properties of figures and can describe steps taken when questioned about geometric procedures. The student recognizes connections between geometry and other disciplines when they are pointed out.

Sample Performance Indicators

The student:
- completes construction tasks in two or three dimensions, given the appropriate tools or materials and some direction;
- identifies and classifies geometric figures and describes their properties;
- exhibits familiarity with geometric terms and concepts;

- solves multi-step problems with some assistance;
- demonstrates some independence in using available geometry-related programs on the computer.

Proficient Performance

The student works independently and co-operatively and seeks assistance when appropriate. The student may choose to use concrete materials as models to clarify understanding of geometric concepts. The student begins to show independence in connecting geometry to other strands of mathematics and other disciplines and intentionally applies geometry in the surrounding world. The student explains geometric procedures so that others can understand.

Sample Performance Indicators

The student:
- completes construction tasks in two or three dimensions accurately, using appropriate tools or materials without assistance;
- identifies and describes geometric properties in precise language;
- exhibits a command of geometric terms and concepts;
- solves a wide variety of problems and investigations in geometric contexts;
- independently uses available geometry-related programs on the computer.

Superior Performance

The student works independently. The student is an abstract thinker and can create and demonstrate geometric relationships in two and three dimensions by experimentation or through simulation. The student connects geometry to other disciplines. The student articulates geometric procedures and explains them to others in mathematical language.

Sample Performance Indicators

The student:
- completes construction tasks in two or three dimensions, accurately and without assistance, using appropriate tools, and suggests alternate methods of construction;
- poses and solves a wide variety of problems and conducts a variety of investigations in geometric contexts.

Measurement

Outcomes From *The Common Curriculum*

Students will:
- identify, develop, and apply procedures for measuring the characteristics of a variety of shapes, liquids, and solids and for examining the interrelationships between and among mass/weight, volume and density, speed/velocity, distance, displacement, and time, using concrete materials and appropriate units. (p. 85)
- value and use mental calculations and the skills of computational and measurement estimation, and be able to apply those skills to new problem–solving tasks. (p. 85)

More specifically, students will:
- understand what is measured by: linear measure, area, volume, capacity, mass, temperature, and time;
- recognize whether a measurement or an estimate is appropriate;
- estimate and use formulas to determine the perimeter and area of rectangles, triangles, parallelograms, and trapezoids;
- construct regular and irregular figures with given perimeters or areas, using a variety of materials;
- determine the relationship between figures with fixed perimeters and varying areas, and vice versa;
- estimate and determine the circumference and area of circles;
- estimate and use formulas to determine the surface area and volume of rectangular and triangular prisms and cylinders;
- estimate and use formulas to determine the surface area and volume of pyramids and cones;
- state and apply the Pythagorean Theorem;
- solve problems involving perimeter, area, surface area, and volume;
- estimate and measure the perimeters and areas of irregular figures, using a variety of techniques;
- demonstrate an understanding of the effect of varying dimensions on area and volume.

Standards of Performance

Limited Performance

The student is very dependent on teacher direction. The student relies on models or given diagrams when making measurement calculations. The student exhibits little understanding of measurement beyond linear measurement and has limited retention of information about relationships between units.

Sample Performance Indicators

The student:
- guesses rather than estimates;
- often leaves out units or uses incorrect units;
- performs a required calculation when given the appropriate formula;
- uses the calculator to perform the necessary calculations, with assistance;
- uses available measurement-related programs on the computer, with assistance;
- solves one-step problems in measurement contexts, with direction.

Adequate Performance

The student is frequently dependent on the teacher or other students for assistance. The student uses concrete materials to establish or clarify understanding of measurement concepts such as surface area and volume. The student appreciates the importance of measurement in day-to-day living. When questioned about measurement activities, the student can describe the steps taken.

Sample Performance Indicators

The student:
– estimates, but needs assistance in judging the reasonableness of the estimate;
– selects the appropriate unit, with guidance;
– uses a formula correctly, but may not be able to explain how the formula was developed;
– solves multi-step problems in measurement contexts with assistance;
– uses the calculator to do calculations and also as a tool for investigation, with assistance;
– demonstrates some independence in using available measurement-related programs on the computer.

Proficient Performance

The student works independently and co-operatively and seeks assistance when appropriate. The student may choose to use concrete materials as models to clarify understanding of surface area and volume. The student begins to show independence in relating measurement to other strands of mathematics and other disciplines and intentionally applies measurement to the surrounding world. The student can give coherent explanations of steps taken in measurement activities.

Sample Performance Indicators

The student:
– estimates and judges the reasonableness of the estimate;
– selects appropriate units and converts from one unit to another with accuracy;
– uses formulas correctly and understands how they were developed;
– solves a wide variety of problems and carries out and draws conclusions from investigations in measurement contexts;
– uses the calculator to solve problems and as a tool for investigating relationships;
– independently uses available measurement-related programs on the computer.

Superior Performance

The student works independently. The student relates measurement to other disciplines and real-life situations. The student articulates the steps taken in measurement activities and explains them to others.

Sample Performance Indicators

The student:
– uses a personal set of referents when estimating;
– selects the appropriate unit and converts from one unit to another with facility and accuracy;
– poses and solves a wide variety of problems and carries out and draws conclusions from investigations in measurement contexts;
– makes appropriate choices of available technology when solving and investigating measurement relationships.

Patterning and Algebra

Outcomes From *The Common Curriculum*

Students will:
- use algebraic notation to represent the general term of a pattern and employ algebraic modelling as a strategy in solving relevant real-life problems. (p. 78)
- use and appreciate the value of algebraic notation in mathematics. (p. 80)
- use co-ordinate geometry to describe and represent relationships defined by tables, graphs, and equations or formulas. (p. 85)
- represent mathematics symbolically. (p. 85)
- solve problems using linear equations with one variable. (p. 85)

More specifically, students will:
- express generalizations about patterns both in words and algebraically, and develop formulas as generalizations about observed patterns;
- use calculators and computers to investigate patterns;
- use patterns to represent and solve problems;
- describe and represent patterns in tables and graphs;
- make predictions based on pictorial representations, tables of values, graphs, and equations;
- solve problems by using tables, graphs, and equations;
- develop the concepts of a variable, an expression, and an equation;
- manipulate algebraic expressions only as necessary to solve equations in one variable;
- model and solve problems using linear equations and inequalities;
- solve linear equations by formal and informal methods;
- substitute into a formula and solve the resulting equation;
- graph relations, including linear relations, using data based on information from everyday life;
- solve equations and graph relations that are relevant to other subject disciplines.

Standards of Performance

Limited Performance

The student is very dependent on teacher direction. The student relies on concrete materials to visualize patterns and has very little understanding of the concept of a variable. The student sees little connection between real-life and algebraic representations.

Sample Performance Indicators

The student:
- continues a variety of patterns derived from number and geometric concepts using a wide variety of materials;
- uses concrete materials for the representation of variables and as an aid in the manipulation of expressions;
- recognizes the mathematical translation of words or phrases;
- solves an equation involving one operation by trial and error or inspection;
- completes a table showing information from an experiment or pattern, with assistance;
- completes a graph showing information from an experiment or pattern, with assistance;
- solves one-step problems with direction.

Adequate Performance

The student frequently depends on other students or the teacher for assistance. The student may require the use of concrete materials to help understand patterns and the manipulation of variables, but is beginning to use formal methods for simplifying expressions. The student uses algebra as a tool to solve problems when directed to do so. The student will normally attempt other strategies in problem–solving situations in which algebraic modelling would be the most effective.

Sample Performance Indicators

The student:
– describes, predicts, and verifies a pattern where concrete modelling is practical;
– translates words and/or phrases modelling real-life situations into mathematical expressions;
– is beginning to employ formal methods to simplify algebraic expressions and to solve equations with more than one operation;
– is beginning to use equations as a strategy to solve problems;
– draws a graph in the first quadrant using data from real life, with some assistance;
– with assistance, uses appropriate technology to graph relations.

Proficient Performance

The student works independently and co-operatively and seeks assistance when appropriate. The student is beginning to use algebraic symbols for communication and realizes that proper form is important. The student sees that algebra is an effective problem–solving tool and uses it appropriately.

Sample Performance Indicators

The student:
– describes, predicts, and verifies a pattern beyond the point where concrete modelling is practical, using a variable;
– employs formal methods to simplify algebraic expressions and to solve linear equations;
– uses equations to solve problems in a variety of contexts;
– draws graphs of linear relations using all four quadrants and uses available technology where appropriate.

Superior Performance

The student works independently. The student relates algebraic procedures to other disciplines. The student consistently uses proper form in solutions.

Sample Performance Indicators

The student:
– uses appropriate patterns to model and solve problems;
– generalizes (extends the application of) the process for solving linear equations in one variable;
– simplifies algebraic expressions beyond what is necessary to solve equations in one variable.

Data Management and Probability

Outcomes From *The Common Curriculum*

Students will:
- gather data from a wide variety of sources, analyse and interpret it, make decisions based on it, and communicate the information and decisions in a fashion appropriate to the intended audience. (p. 94)
- investigate and apply the concepts of probability and statistics in the course of inquiries. (p. 94)
- use calculators and computers to explore patterns and relationships and solve problems relating to regional, national, and global settings. (p. 95)

More specifically, students will:
- using graphs from various sources, classify them according to their type and identify their characteristics and uses;
- read charts, tables, and graphs and interpret the information using both interpolation and extrapolation;
- construct graphs from a variety of sources and pose and answer questions about the graphs constructed;
- identify the various methods used to collect data and state the advantages and disadvantages of each method;
- establish a plan and procedure for collecting data through surveys or experiments, collect the data, analyse, organize, interpret, and display the information appropriately, and pose and answer questions about the data collected;
- determine mean, median, and mode from collected data and use them to analyse the data appropriately;
- estimate populations using sampling techniques and explain why predicted results do not always match actual results;
- predict outcomes by creating and carrying out a probability experiment or simulation to model a problem;
- determine probabilities of events from their relative frequencies, using information from individual and class investigations;
- collect examples of and comment on the use of probability in the media.

Standards of Performance

Limited Performance

The student is very dependent on teacher direction. The student is most comfortable with small amounts of data and short precise instructions. With prompting, the student recognizes applications of statistics and probability in the surrounding world and uses these to help make decisions.

Sample Performance Indicators

The student:
- collects, organizes, and displays data when given specific instructions and answers teacher-directed questions;
- carries out a probability experiment for which the instructions are provided and records the results.

Adequate Performance

The student frequently depends on other students or the teacher for assistance. The student follows a series of instructions, makes some connections among data, and does some interpretation. The student recognizes data management and probability as areas of mathematics that are part of everyday life. The student often accepts media interpretations of data without question.

Sample Performance Indicators

The student:
– collects, organizes, and displays data when given instructions and analyses and interprets the results, with some teacher direction;
– carries out a probability experiment for which the instructions are provided and records and analyses the results, with some teacher assistance;
– with assistance, uses appropriate available technology to organize and display data.

Proficient Performance

The student works independently and co-operatively and seeks assistance when appropriate. The student will often independently identify and take the necessary steps to complete tasks in data management and probability. The student is aware of and independently uses available referents to gather data, statistics, and probability information, when appropriate. The student is able and willing to question media interpretations of data and to offer his or her own interpretation. The student transfers learned techniques and concepts of data management and probability into other content areas.

Sample Performance Indicators

The student:
– identifies a question of personal interest or one related to a social issue and collects, organizes, displays, and analyses data related to it;
– makes use of available technology in collecting, organizing, displaying, and analysing data;
– carries out a probability experiment and draws conclusions.

Superior Performance

The student works independently. The student establishes a framework from which to investigate an issue at the school level and beyond. The student independently creates probability experiments, draws conclusions, and makes some theoretical connections.

Sample Performance Indicators

The student:
– poses a question, investigates it, creates a model, and makes informed statements about the conclusion;
– poses a problem and then designs, investigates, carries out, and evaluates a probability experiment or simulation;
– uses available technology to help organize and display data.

Appendix A

Outcomes From *The Common Curriculum* That Deal With Underlying Processes in Teaching and Learning Mathematics

Some outcomes in *The Common Curriculum* do not relate directly to the content of a particular strand or a key component of mathematics, but are highly relevant to the classroom environment and the teaching/learning strategies used. These outcomes deal with the underlying processes of teaching and learning mathematics, and it is important for teachers to take them into account in planning lessons and evaluating student performance.

Outcomes From *The Common Curriculum*

Grade 3

Students will:
- take note of and evaluate the views, opinions, and cultural perspectives of people with whom they interact during inquiries and while collecting and using data. (p. 81)
- identify examples from their own and other cultures that demonstrate the contributions to mathematics, science, and technology of people of both genders and from a variety of racial/cultural backgrounds. (p. 82)
- use safely a variety of simple tools and manipulative materials to extend the senses in hands-on investigations. (p. 82)
- use concrete materials, computers, and other tools and equipment to construct representations of simple concepts and procedures in mathematics, science, and technology. (p. 83)
- understand that mathematics, science, and technology involve the use of manipulative materials, tools, equipment, resources, and processes and give examples of such applications. (p. 83)
- use mathematical, scientific, and technological skills and knowledge in the classroom and in their daily lives. (p. 89)
- identify ways in which mathematics, science, and technology are used in the world around them. (p. 93)
- demonstrate a bias-free awareness of different types of careers involving mathematics, science, and technology and describe ways these careers meet various needs in the community. (p. 93)
- perceive and acknowledge the importance of applying themselves in mathematics, science, and technology activities. (p. 93)
- demonstrate a positive attitude towards mathematics, science, and technology; manipulate concrete materials spontaneously and with confidence; and show respect for the success and contributions of others. (p. 94)

Grade 6

Students will:

- take note of a variety of views, opinions, and cultural perspectives and evaluate them before making judgements, bearing in mind that many currently held views reflect a Eurocentric perspective. (p. 81)
- identify, compare, and discuss examples from a variety of cultures that demonstrate the contributions to mathematics, science, and technology of people of both genders, and from a variety of racial/cultural backgrounds. (p. 82)
- use safely a variety of tools, equipment, and manipulative materials to support personal and group inquiry, experimentation, and information gathering and sharing. (p. 82)
- use concrete materials, computers, and other tools and equipment to construct representations of increasingly complex concepts and procedures in mathematics, science, and technology. (p. 83)
- recognize that mathematics, science, and technology involve the application of concepts. (p. 83)
- use mathematical, scientific, and technological skills and knowledge in the classroom, school, and their daily lives, and explain the importance of such knowledge and skills to society. (p. 89)
- describe the roles played by mathematics, science, and technology in a variety of career and work opportunities. (p. 93)
- understand that careers involving mathematics, science, and technology are accessible to all who are willing to prepare for them. (p. 93)
- perceive a connection between their personal effort and self-discipline and their achievement in the mathematics, science, and technology program. (p. 93)
- demonstrate a positive attitude towards mathematics, science, and technology; confidently and spontaneously conduct an inquiry involving the manipulation of concrete materials; and show respect for the success and contributions of others. (p. 94)

Grade 9

Students will:

- examine and objectively evaluate a variety of perspectives before making judgements during inquiries and research. (p. 81)
- identify, compare, and evaluate objects, processes, and systems drawn from a variety of cultures in order to demonstrate the contributions to mathematics, science, and technology of people of both genders and from a variety of racial/cultural backgrounds. (p. 82)
- use safely and evaluate a variety of tools, equipment, and manipulative materials to support personal and group design, construction, inquiry, experimentation, information gathering, research, and independent studies. (p. 82)
- use concrete materials, computers, instruments, and other tools and equipment to construct representations of concepts and procedures in mathematics, science, and technology. (p. 83)
- understand, select, and use a broad range of mathematical, scientific, and technological concepts and skills in inquiry and design activities. (p. 83)
- understand that learning in mathematics, science, and technology occurs through the process of overcoming obstacles and solving problems. (p. 84)

- use mathematical, scientific, and technological skills and knowledge to investigate a variety of issues, and demonstrate an awareness of the benefits and limitations of such knowledge and skills. (p. 89)
- examine their own interests, aptitudes, and abilities in relation to a variety of career and work opportunities and investigate the role of mathematics, science, and technology in these occupations. (p. 93)
- use statistical information to identify groups of people who may be under-represented in careers in mathematics, science, and technology, and discuss ways of eliminating such inequities. (p. 93)
- establish personal goals and work towards achieving them in order to maximize their learning in mathematics, science, and technology. (p. 93)
- demonstrate a positive attitude towards mathematics, science, and technology; confidently engage in independent inquiry, research, and the manipulation of concrete materials; and show respect for the success and contributions of others. (p. 94)
- use computers to develop, explore, and evaluate simulations/models of real or imaginary situations, products, systems, and processes. (p. 95)

Appendix B

Assessment and Evaluation in the Mathematics Classroom[5]

Outcomes-based education stresses process. Outcomes are the observable/measurable knowledge, skills, and values that students are expected to have acquired and developed at certain key stages of their schooling. They describe what students should know, should be able to do, and should value as a result of their learning experiences. Content becomes the tool for attaining outcomes. This shift toward outcomes-based education requires a modification of assessment practices.

Increased attention should be given to a broad range of assessment practices such as:
– assessing what students know and how they think about mathematics;
– focusing on a broad range of mathematical tasks and taking a holistic view of mathematics;
– assessing student performance in a variety of ways, including written, oral, and demonstration forms;
– using calculators, computers, and manipulatives;
– recognizing such attitudinal outcomes as motivation and appreciation;
– assessing process as well as product.

Tests are one way of determining what students have learned. But mathematical competence involves such characteristics as communicative ability, problem–solving ability, higher-order thinking ability, creativity, persistence, and curiosity. A variety of assessment strategies should be used to provide students with opportunities to demonstrate these characteristics.

The following assessment strategies are recommended in order to supplement the traditional pencil-and-paper testing: performance assessment, open-ended questions, investigations, journals, observations, conferences and interviews, portfolios, and student self-assessment.

Performance Assessment

Performance assessment in mathematics involves presenting students with a mathematical task, project, or investigation, then observing them, interviewing them, and examining what they produce to assess what they actually know and can do. Performance assessment focuses on the process as well as the product. It provides contexts that have relevance to the students' lives and enhances motivation as students work on real tasks.

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5. The content of this section is adapted from the document *Focus on Renewal of Mathematics Education.*

Performance assessment gives information about a student's ability to:
– use mathematical concepts and skills;
– reason soundly and raise questions;
– think flexibly, changing strategies when a particular approach does not work;
– use manipulative materials and equipment, calculators, and computers;
– work together in groups;
– persist, concentrate, and work independently;
– communicate and use mathematical language through discussing, writing, and explaining their ideas and questions;
– observe carefully to infer and formulate hypotheses;
– design and conduct experiments and investigations.

Open-Ended Questions

"Open-ended" means having no predetermined limit or boundary. Students are asked to respond to questions or solve problems to which a variety of successful responses are possible.

Open-ended questions give information about a student's ability to:
– recognize the essential points of the problem;
– organize and interpret information;
– make generalizations;
– write for a given audience;
– understand basic concepts;
– use appropriate mathematical language and representation;
– demonstrate originality and/or creativity.

Investigations

Investigations involve explorations of mathematical questions that may be related to other subject areas. Investigations deal with problem posing as well as problem solving.

Investigations give information about a student's ability to:
– identify and define a problem;
– make a plan;
– create and interpret strategies;
– collect and record needed information;
– organize information and look for patterns;
– persist, looking for more information if needed;
– discuss, review, revise, and explain results.

Journals

A mathematics journal is a personal, written expression of thoughts. Students express ideas and feelings, ask questions, draw diagrams and graphs, explain processes used in solving problems, report on investigations, and respond to open-ended questions.

Journals allow opportunities for students to:
– formulate, organize, internalize, and evaluate concepts;
– clarify thinking;
– identify their own strengths, weaknesses, and interests;
– reflect on new learning;
– use the language of mathematics to describe learning.

The information provided by journals can be useful to teachers in modifying the program to meet individual needs.

Observations

Research has consistently shown that the most reliable method of evaluation is the ongoing, in-class observation of students by teachers. Students should be observed as they work individually and in groups.

Systematic, ongoing observation gives information about students':
– attitudes toward mathematics;
– feelings about themselves as learners of mathematics;
– specific areas of strength and weakness;
– preferred learning styles;
– areas of interest;
– work habits;
– social development;
– development of mathematics language and concepts.

In order to ensure that the observations are focused and systematic, a teacher may use checklists, a set of questions, and/or a journal as a guide.

Teachers should develop a realistic plan for observing students. The plan might incorporate opportunities to:
– observe a small number of students each day;
– focus on one or two aspects of development at a time;
– tape-record or videotape student activities for later analysis.

Conferences and Interviews

An interview includes a planned sequence of questions, whereas a conference implies discussion, with student and teacher sharing ideas. The main purpose of both conferences and interviews is to explore the student's mathematical thinking and to assess the student's level of understanding of a particular concept or procedure.

Conferences and interviews give information about students' ability to:
– explain the process used to arrive at a solution;
– justify their thinking (i.e., explain and support their reasoning);
– suggest an alternative strategy;
– discuss their likes and dislikes;
– discuss their perceived strengths and weaknesses.

Techniques that contribute to a successful conference/interview include:
– using prepared questions;
– putting students at ease;
– explaining that you are interested in the process of reasoning followed by the student;
– posing a problem;
– taking notes;
– being a good listener;
– being non-judgemental;
– doing instructional intervention in a separate setting.

Most conferences/interviews should be brief and informal and should occur naturally in the course of everyday mathematical experiences.

Portfolios

A portfolio is a showcase for student work, a place where many types of work can be collected. The portfolio provides a comprehensive view of the student's progress in, attitudes toward, and understanding of mathematics.

A portfolio may include the following examples of student work:
– written descriptions of investigations;
– descriptions and/or analyses of problem situations;
– responses to open-ended questions;
– homework problems;
– photographs of a project;
– group reports;
– video, audio, or computer-generated examples of work;
– personally chosen work, with reasons for the choice;
– excerpts from the student's mathematics journal;
– a description of how the student feels about mathematics;
– a student self-report on what has been learned.

Student portfolios can provide:
– evidence of performance;
– evidence of growth or change over a period of time;
– a permanent record of student performance;
– opportunities for students to practise assessing and selecting their own work.

Student Self-Assessment

Student self-assessment promotes the development of metacognition skills (the ability to reflect critically on one's own reasoning). It also assists students to take ownership of their learning, and become independent thinkers. Self-assessment can be done by means of a questionnaire following a co-operative activity or project, asking how well the group functioned, or through daily writing in a journal.

Teachers can use student self-assessments to determine:
– whether there is change and growth in the student's attitudes, mathematical understanding, and achievement;
– whether a student's beliefs about his or her performance correspond to actual performance;
– whether the student and the teacher have similar views of expectations and criteria for evaluation.

Appendix C
Resources Used

The following resources have been used in the preparation of sections of this document:

Working Groups of the Commission on Standards for School Mathematics. *Curriculum and Evaluation Standards for School Mathematics.* Reston, Va.: National Council of Teachers of Mathematics, 1989.

Ontario Association for Mathematics Education (OAME) and Ontario Mathematics Coordinators Association (OMCA). *Focus on Renewal of Mathematics Education.* Toronto: OAME and OMCA, 1993.